ADDENDA TO:

THE LINE OF RESISTANCE

Dr. Lawrence D. Woolf
General Atomics
San Diego, CA 92121
Larry.Woolf@gat.com
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ADDENDUM TO 1A: ELECTRICAL RESISTIVITY OF GRAPHITE

There is another way to determine the electrical resistivity of the graphite in the graphite pencil: utilize the pencil itself instead of using a line drawn with the graphite pencil. At the ends of the pencil, scrape off the insulating outer coating of the pencil to expose the underlying graphite. (On some pencils this will not be necessary – use the sharpened writing end and the exposed graphite at the center of the other end.) Using the ohmmeter, measure the electrical resistance of the graphite in the pencil by applying the probes to the exposed graphite at the ends of the pencil. Here, the length L is the distance between probes of the ohmmeter, which is essentially the length of the pencil. Since the cross section R area of the pencil is πr^2, then the electrical resistivity ρ of the graphite can be determined using:

$$\rho = RA/L = R\,(\pi r^2)/L.$$

If R=3.5 ohms, r = 0.3 cm, and L = 13 cm, then

$$\rho = \frac{(3.5 \text{ ohms})\,(3.14)\,(0.3 \text{ cm})^2}{13 \text{ cm}} = 0.076 \text{ ohm-cm}$$

Note that this experiment will provide a direct measurement of ρ and will also test the student's understanding of the cross sectional area, which is circular in this case, and rectangular the case of the drawn line. Note that the cross section is the area through which current flows from one probe to the other probe. Using the value of ρ determined above, the thickness of the drawn line can now be determined using the above equation. Recall that the cross sectional area of the line is given by A=wt, where w is the width of the line and t is the thickness of the line.

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$A = wt = \rho L/R$, so the thickness of the line can be calculated using:

$t = \rho L/(Rw)$.

If L = 5 cm, w = 0.5 cm, ρ = 0.076 ohm-cm, and R=30,000 ohms, then

$$t = \frac{(0.076 \text{ ohm-cm})\ (5 \text{ cm})}{(30{,}000 \text{ ohm})\ (0.5 \text{ cm})} = 2.5 \times 10^{-5} \text{ cm} = 0.25 \text{ micrometers}$$

So the thickness of the line is calculated to be 0.25 micrometers. The teacher should note that a number of critical assumptions were made. First, we assumed that the electrical resistivity of the graphite in the pencil is the same as the electrical resistivity of the graphite in the line. This assumption may not be valid because the density and packing of the graphite in the pencil may be different than that of the graphite in the line. Second, we assumed that the contact resistance of the pencil was negligible. For the pencils tested, the contact resistance of the graphite in the pencil is comparable to the bulk resistance.

Scanning electron micrographs were obtained by viewing a pencil line at an angle, as shown below at three different magnifications: 400X, 2000X, and 10,000X. In the 10,000X magnification photo (note the 2 micrometer (μm) scale marker), it is apparent that the thickness of the line, or at least many of the flakes, is about 0.25-0.50 microns, in reasonable agreement with the above calculation.

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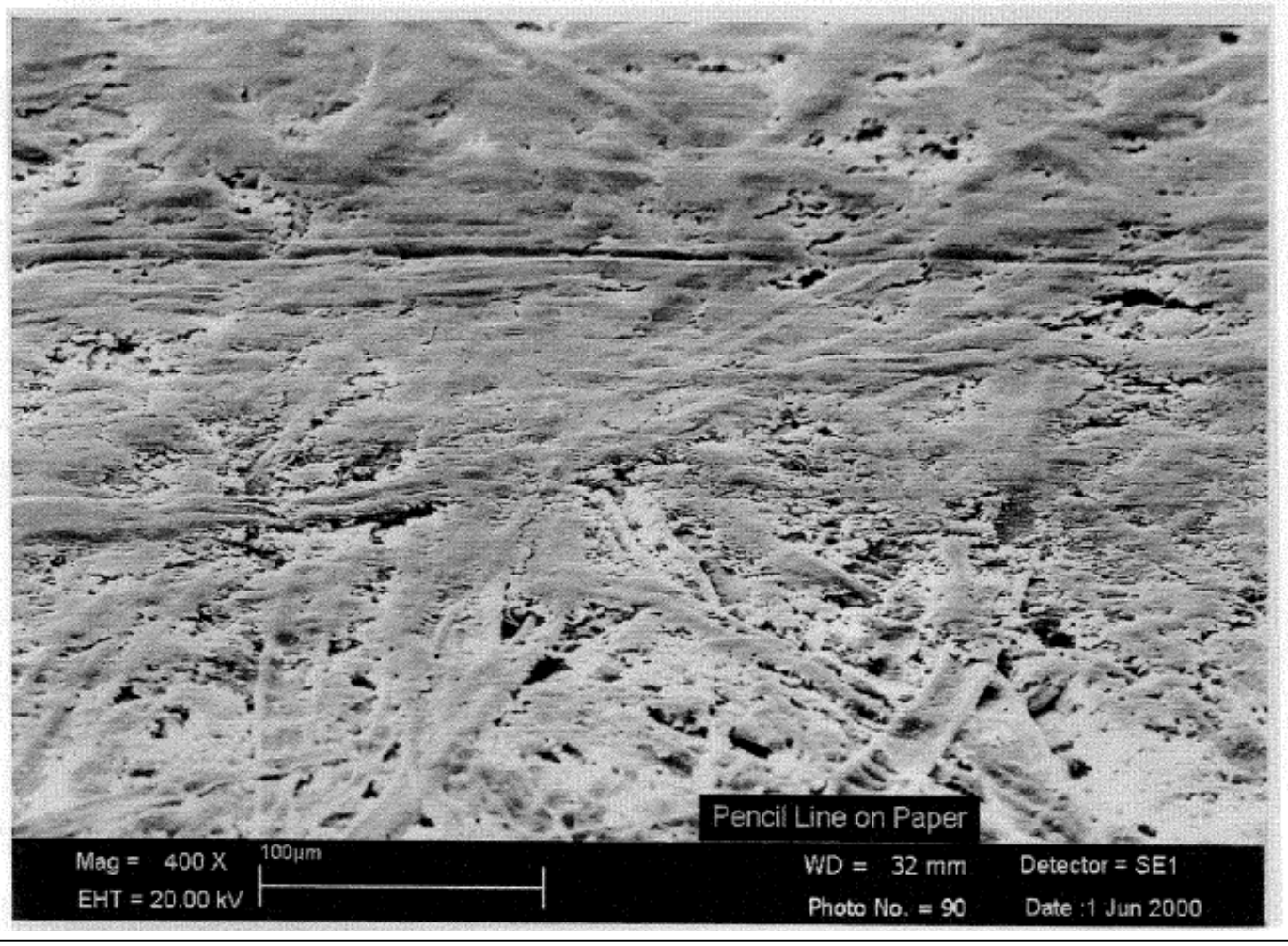

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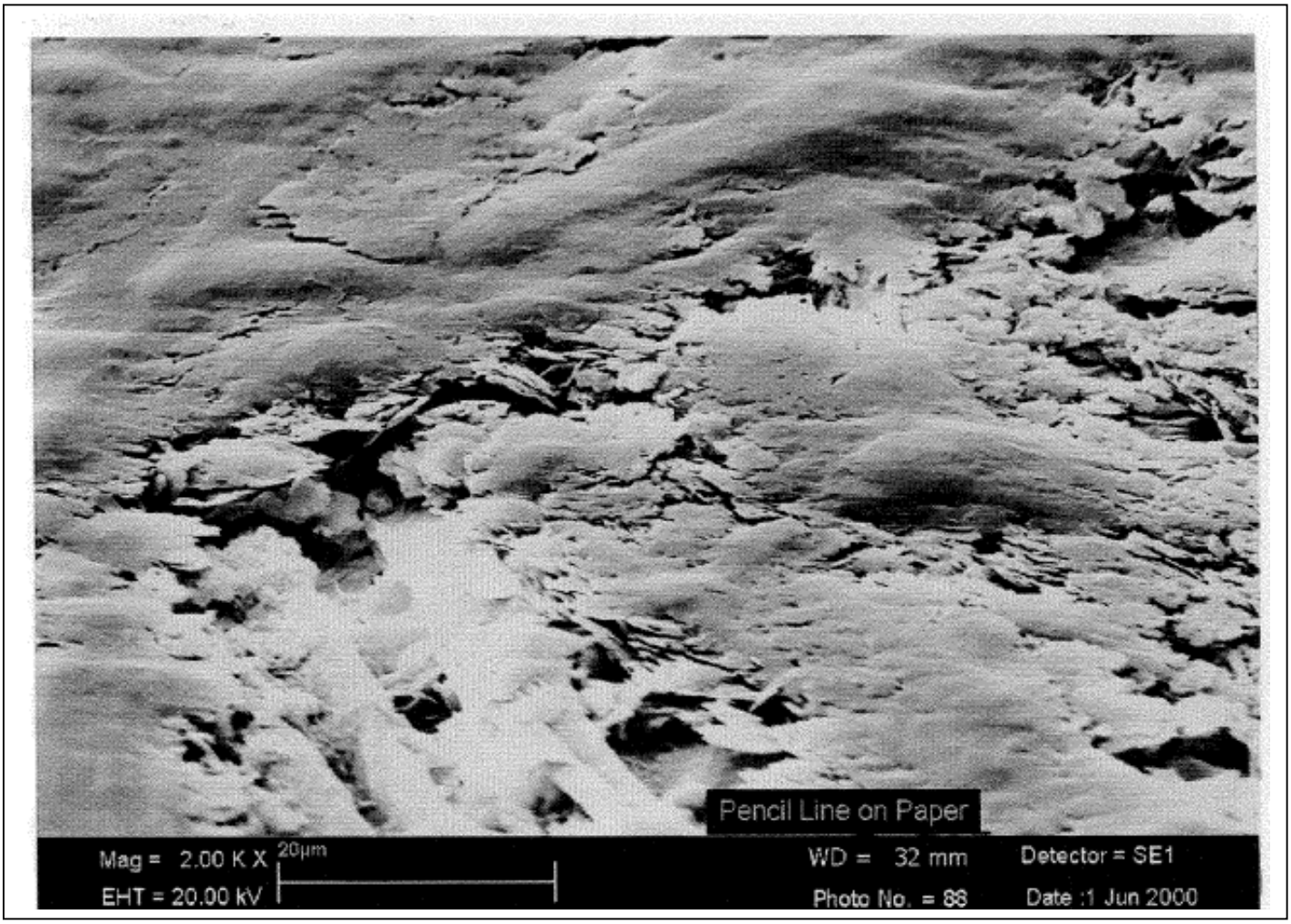

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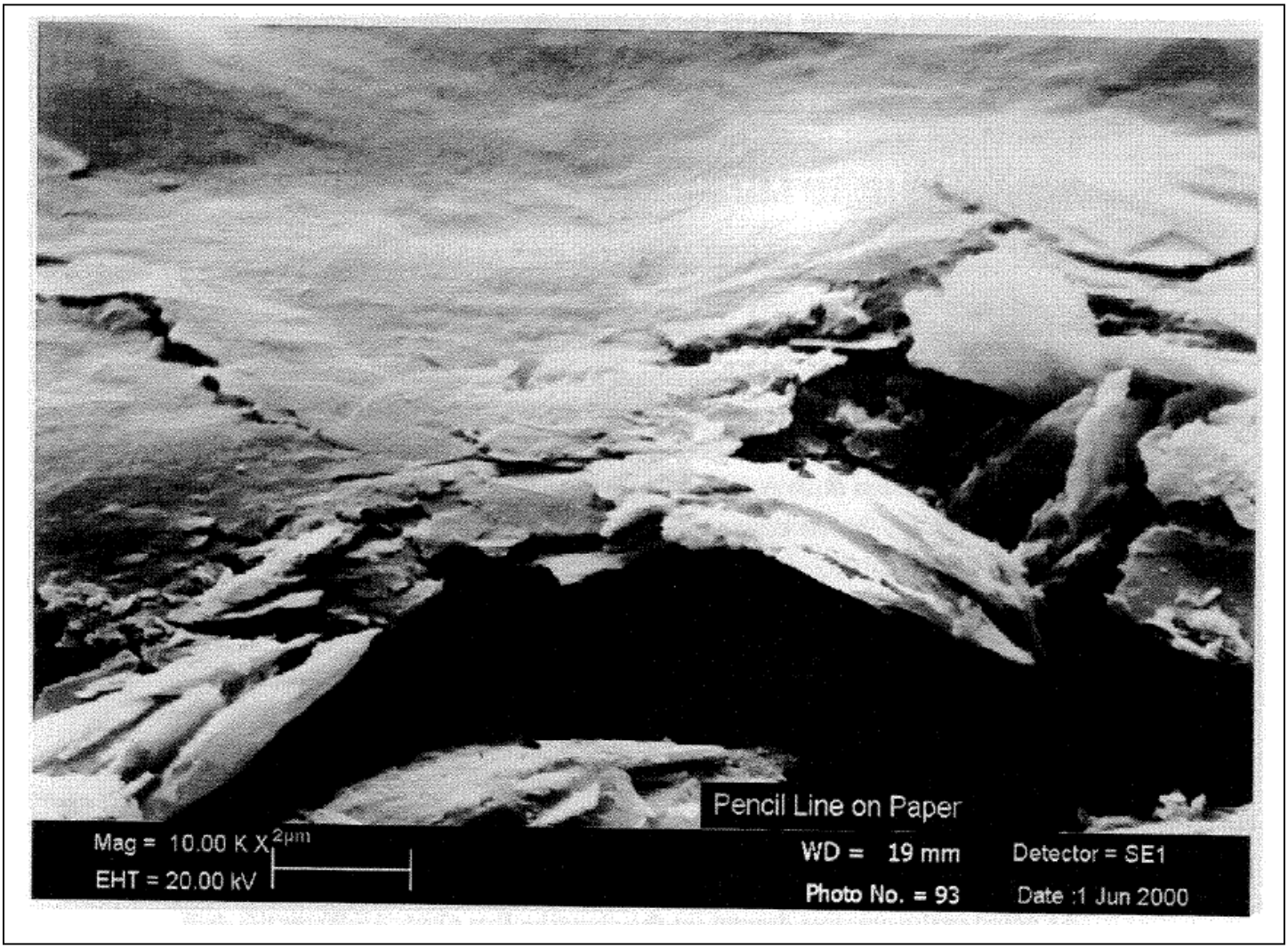

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ADDENDUM TO IIA: RESISTANCE OF SERIES CIRCUITS

A better way to demonstrate mathematically that equation 12 is really just a restatement of equations 1 is as follows:

Recall that $R = \rho L/A$ and consider the case where the cross sectional area A is fixed.

Consider resistor 1 with length L_1: $R_1 = \rho L_1/A$

Consider resistor 2 with length L_2: $R_2 = \rho L_2/A$

Now consider a resistor with length $L_1 + L_2$:

$$R_{total} = \rho(L_1 + L_2)A = \rho L_1/A + \rho L_2/A = R_1 + R_2$$

So the electrical resistance of a resistor with length $L_1 + L_2$ is the same as the sum of the electrical resistance of a resistor with length L_1 and a resistor of length L_2.

ADDENDUM TO IIB: RESISTANCE OF PARALLEL CIRCUITS

A better way to demonstrate mathematically that equation 13 is really just a restatement of equations 1 is as follows:

Recall that $R = \rho L/A$ and consider the case where the length L is fixed.

Consider resistor 1 with cross sectional area A_1: $R_1 = \rho L/ A_1$

Note that $1/ R_1 = A_1/(\rho L)$

Consider resistor 2 with cross sectional area A_2: $R_2 = \rho L/ A_2$

Note that $1/ R_2 = A_2/(\rho L)$

Now consider a resistor with cross sectional area $A_1 + A_2$:

$$R_{total} = \rho L/ (A_1 + A_2)$$

So $1/R_{total} = (A_1 + A_2)/(\rho L)$

Or $1/R_{total} = A_1/(\rho L) + A_2/(\rho L) = 1/ R_1 + 1/R_2$

So the 1/(electrical resistance) of a resistor with cross sectional area $A_1 + A_2$ is the same as that of the sum of 1/(electrical resistance) of a resistor with cross sectional area A_1 + 1/(electrical resistance) of a resistor with cross sectional area A_2.

As noted in the creative dramas, in the equation $R = \rho L/A$, L affects the drift speed of the electrons through the resistor: when a constant voltage is applied over a longer distance L, the drift speed of the electrons will slow proportionately. The cross-sectional area A affects the area through which current can flow: the larger the area, the more electrons that participate in the conduction of electricity.

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VI. ELECTRICAL BREAKDOWN

C. Lightning Safety

A rather dramatic example of the process of electrical breakdown is the spectacular phenomenon of lightning. Lightning kills more people each year than hurricanes or tornadoes. Most people injured or killed by lightning are struck while outside. Two important safety tips can be investigated using the piezoelectric gas ignitor and graphite pencil.

1. Consider a person standing up and another person crouching down. Design and perform an experiment to determine which person is more likely to be hit in a lighting storm. What safety tip did you learn from this?

2. The electrical conductivity of the ground can vary greatly, depending on the type of dirt, rocks, moisture, proximity to underground metal pipes, etc. Suppose that lightning strikes the ground near you and there is a highly conducting pathway from that point to where you are. Your feet are straddling a non-flat rock which is very insulating compared to the surrounding ground, as shown in the figure. (Place the feet slightly off of the ground to simulate slightly insulating shoes – it will also allow you to see the path of the lightning.) Determine if the lightning will pass through your body. Now draw a person with their feet touching and see what happens.

a. What safety tip did you learn from this?

b. How is this similar to the lightning rod experiment?

3. a. The breakdown field of moist air is significantly less than that of dry air. Using an electrical breakdown field of moist air of 1000 V/cm and a typical height (use 1-km) for the bottom of a storm cloud, determine the voltage difference between the cloud and the ground during a lightning strike.

b. The average power generated by a lightning strike is 10^{12} W. Determine how much current is flowing from the cloud to the ground.

c. A typical lightning strike lasts for about 30 microseconds. Determine the amount of energy produced by a typical lightning strike.

d. A home may use 500 kW-hr of electrical energy each month. If you could use all of the energy in a lightning strike for that home's electrical supply, how many lightning strikes would be needed?

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ANSWERS:

1. A picture should be drawn of a person standing on the ground just a few mm away from a cloud. The crouching person should be drawn much further away from the cloud. The picture should be similar to one demonstrating the lightning rod.

During a lightning storm, you should not stand up; instead you should crouch down in the lowest possible spot.

2. When straddling the rock, lightning will pass through your body (if you drew the rock to be about 5 mm wide with the person's feet about 1 mm off the ground (representing the insulating soles of their shoes)). With their feet together on the ground, the lightning does not pass through their body because the path through the ground is shorter.

a. During a lightning storm, you should put your feet together, in addition to crouching down.
b. As in the lightning rod experiment, the lightning will travel along the shortest path length between the two electrodes of the lightning generator (the piezoelectric transducer). In the case where the person is straddling the insulating rock, the lightning will travel through the person if L1 + L2 < L3 (see Fig. C2a). In the case where the people have their feet together, the lightning will travel through the rock if L3 < L1 + L2 (see Fig. C2b).

3a. The cloud is at a height of 1km (=1000 m = 10^5 cm). Therefore
V = E x L = 1000 V/cm x 10^5 cm = 10^8 V = one hundred million volts.

3b. I = P/V = 10^{12} W/(10^8 V) = 10,000 A

3c. Energy = Power x time = 10^{12} W x 30 x 10^{-6} sec
= 30 x 10^6 W-sec
= 30 x 10^3 kW-sec x (1 hr/3600 sec)
= 8.3 kW-hr

3d. (1 lightning strike/ 8.3 kW-hr) x 500 kW-hr = 60 lightning strikes

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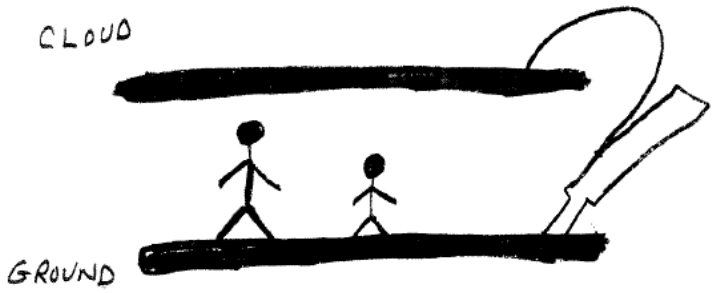
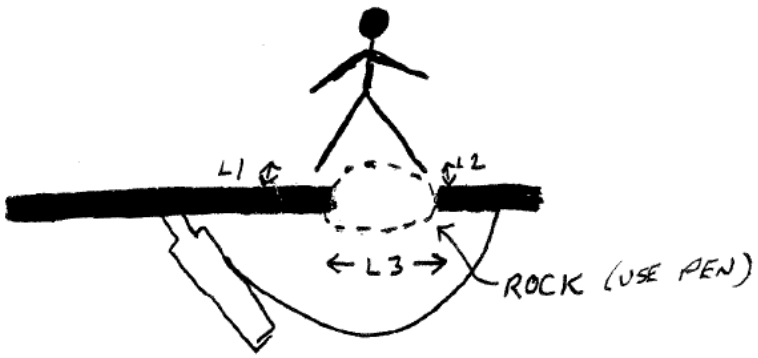
C1.

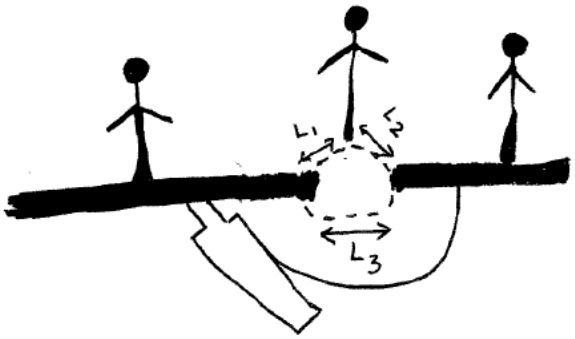


C2.

a.



b.



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VII: SUPERCONDUCTOR SIMULATION

While the phenomenon of superconductivity only occurs at very low temperatures, the principle of superconductivity can be demonstrated at room temperature using the materials contained in this module.

Superconducting wire has no electrical resistance when it is cooled down to a temperature where it is in the superconducting state. So an ohmmeter hooked up to a superconductor would indicate a resistance of zero. If we want to model a superconducting material, then we would need to find a material whose resistance is zero when measured by an ohmmeter. It turns out that the electrical resistance of a piece of copper wire is so low that a typical ohmmeter will indicate that its resistance is zero. So a piece of copper wire at room temperature can be used to simulate a piece of superconducting wire that is at very low temperature. The reason is that a standard laboratory ohmmeter measures any resistance below about 0.5 ohm as zero. A typical copper wire has a resistance of less than 0.1 ohm.

In contrast, a graphite line "wire" has a comparatively large and easily measured electrical resistance. So it can be used to simulate a "normal" conductor, where by "normal" we mean non-superconducting.

In science, this is called a simulation. Simulations are useful for exploring complex phenomena, particularly when it is much easier to generate the simulated phenomena than the real phenomena. The simulation can be summarized as shown in the table below.

	Reality	Simulation
Normal wire (significant electrical resistance)	Copper wire (Truly has measurable resistance)	Graphite line wire (our ohmmeter shows it has electrical resistance)
Superconducting wire (no electrical resistance)	Superconducting wire (Truly has no measurable electrical resistance)	Copper wire (our ohmmeter shows it has no measurable electrical resistance)

1. Fabricate and test normal wire

a. Using the graphite pencil, draw a line on a piece of paper that is about 5 mm wide and about 18 cm long. Cut out the line. Using the ohmmeter, measure the resistance of this tape-shaped "normal" graphite wire.

b. Calculate the cross sectional area of the tape-shaped graphite wire.

2. Fabricate and test superconducting wire

a. What diameter of round superconducting wire would have the same cross sectional area as the tape-shaped normal graphite wire?

b. Cut off a piece of "superconducting" copper wire of approximately this diameter (clean it first with some fine sandpaper or steel wool to remove any insulating

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oxide that may have formed on it) that is also about 18 cm long. Using the ohmmeter, measure the resistance of this round "superconducting" wire that has the same length and cross sectional area as the normal graphite tape-shaped wire.

3. Fabricate normal magnet

Using your normal tape-shaped graphite wire, fabricate a magnet by wrapping 6 turns of this wire around a number 2 pencil. Tape the wire to the number 2 pencil, but keep the ends of the wire free. Suppose you wanted to produce a magnetic field using this magnet that required an current of 1A. How much power would be needed to produce this magnetic field?

4. Fabricate superconducting magnet

Using your "superconducting" copper wire, fabricate a magnet by wrapping 6 turns of this wire around a number 2 pencil. Tape the wire to the number 2 pencil, but keep the end of the wire free. Suppose you wanted to produce a magnetic field using this magnet that required an current of 1A. How much power would be needed to produce this magnetic field? Compare the amount of power a superconducting magnet requires to produce a certain magnetic field compared to a magnet made with non-superconducting materials.

5. (Advanced) Fabricate superconducting magnet with persistent switch.

a. A real low temperature superconducting magnet coil must be connected to non-superconducting materials in order to connect the coil to the power supply. Using the copper "superconducting" wire, number 2 pencil, and graphite "non-superconducting" material, fabricate a complete circuit from the power supply to the superconducting magnet. Just draw a picture of a power supply with positive and negative terminals using a non-electrically conducting writing instrument such as a pen. Electrically connect the "superconducting" wire to the graphite line wire by overlapping a significant portion of the two types of wire and then taping the "superconducting" wire to the graphite line. You may have to experiment some to make a good connection. Show which parts of the system must be maintained at low temperature.

b. Note that when the superconducting magnet is operational, some current flows through the normal material. For the circuit you have constructed, determine how much power is needed to force 1A of current through the superconductor.

c. Consider the current to be flowing in the circuit. Can you figure out a way to modify the circuit by adding a switch so that the current will only flow in the superconducting coil. If you design the switch correctly, the power supply can be turned off, but current will still be flowing in the superconducting magnet!

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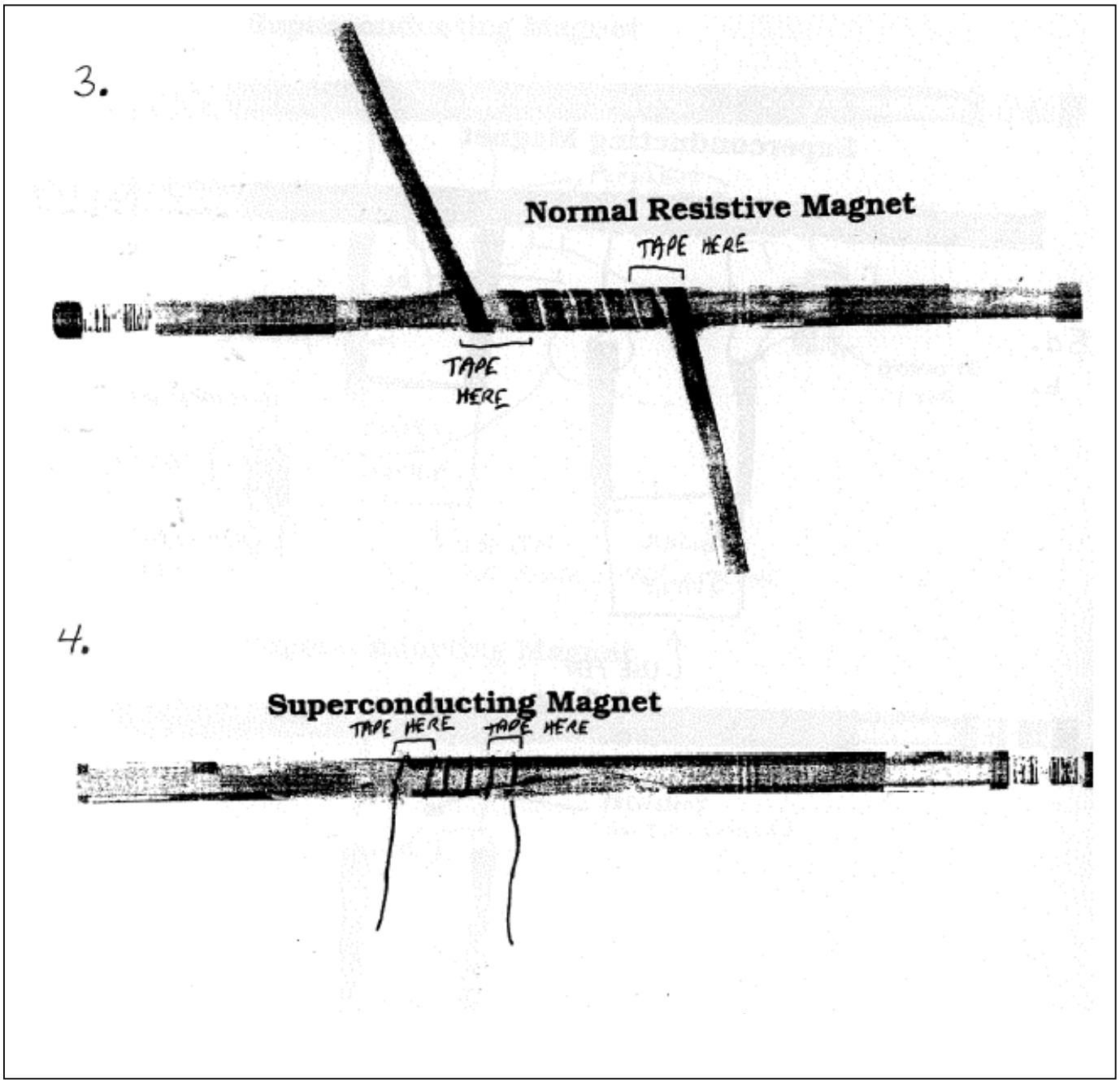

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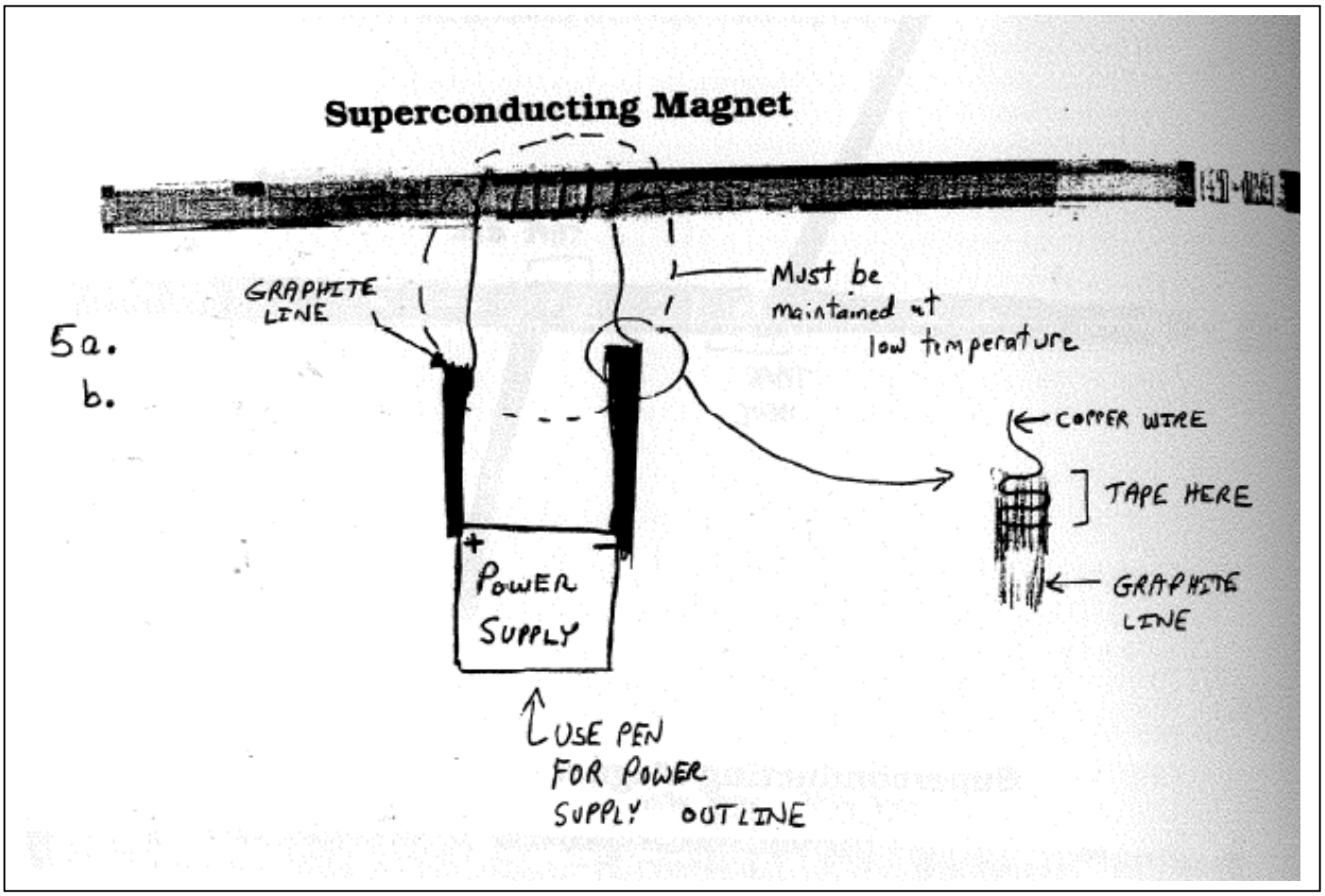

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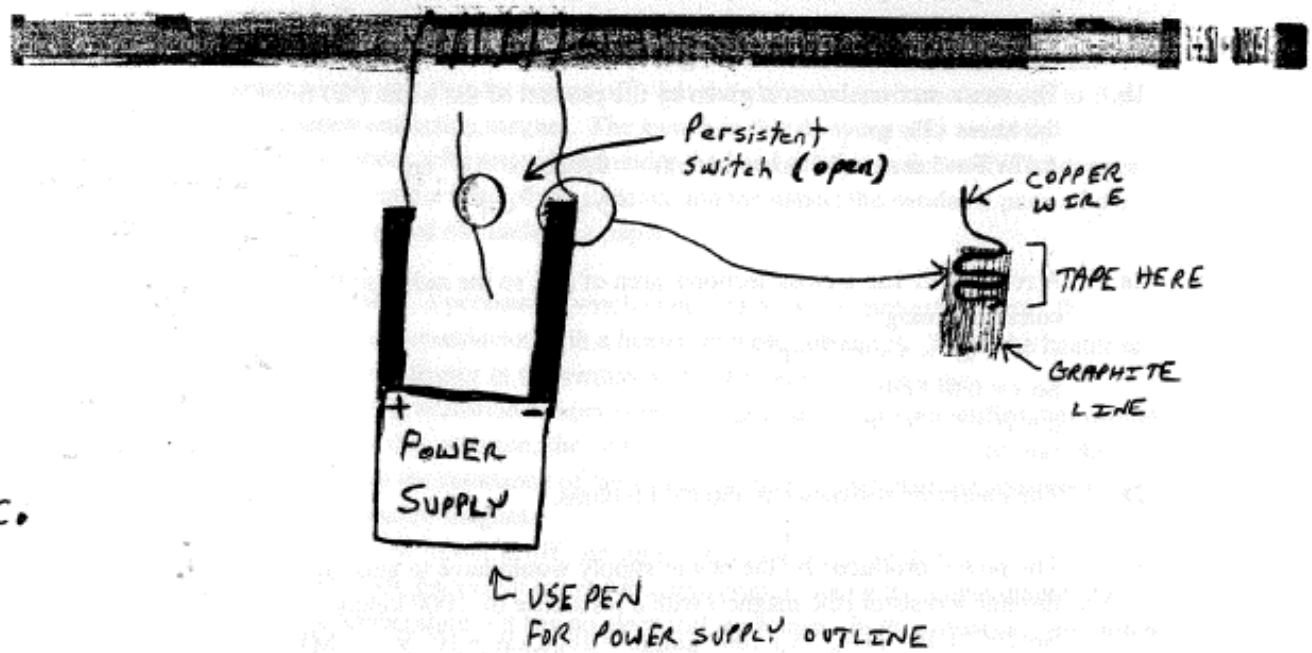
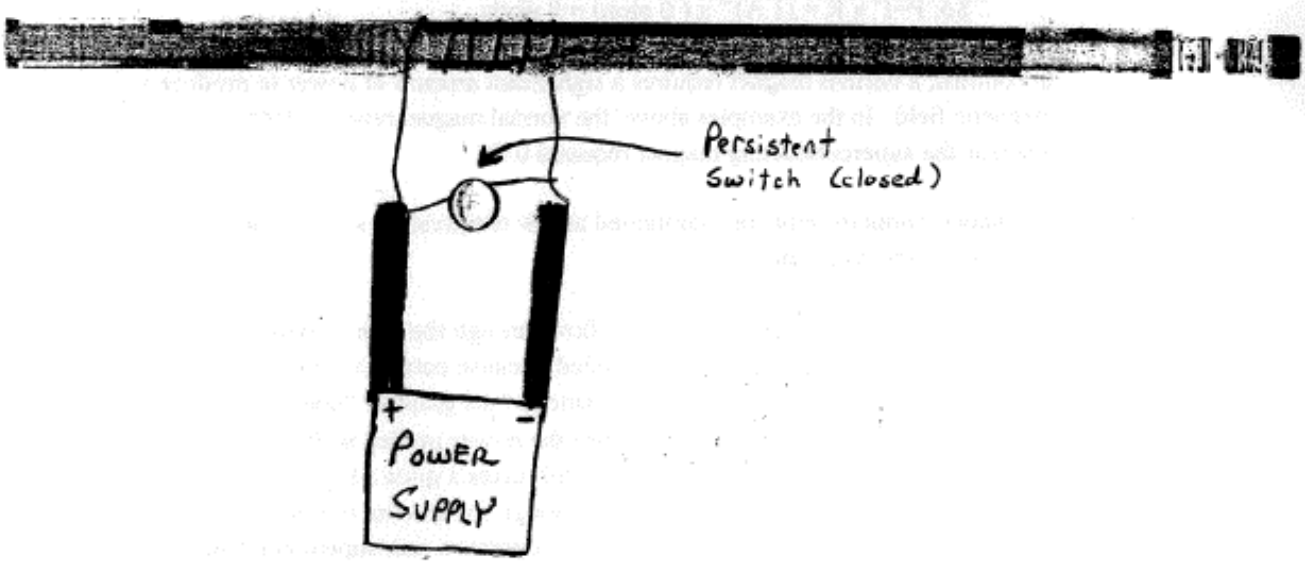
Superconducting Magnet



5c.

Superconducting Magnet



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ANSWERS

1a. The measured resistance is about 1000 kohms = 1 Mohm.

1b. The cross sectional area is given by the product of the width (W) times the thickness (T), so
A= WT = 5 mm x (10 micrometers) $= 0.5 \text{ cm} \times 10^{-3} \text{ cm}$
$= 5 \times 10^{-4} \text{ cm}^2$

2a. A round wire has a cross sectional area of πr^2 , so the radius of the wire can be calculated using:
$\pi r^2 = 5 \times 10^{-4} \text{ cm}^2$
So r = 0.013 cm.
d = 2r = 0.026 cm = 0.26 mm

2b. The measured resistance is about 0.00 ohms.

3. The power produced by the power supply would have to generate 1 A of current through a resistor (the magnet) with a resistance of 1000 kohms.
So: $P=I^2 \times R = (1 \text{ A})^2 \times (1000 \text{ kohm}) = 1000 \text{ kW} = 10^6 \text{ W} = 1 \text{ MW}$.

4. The power produced by the power supply would have to generate 1 A of current through a resistor (the magnet) with a resistance of 0 ohms.
So: $P=I^2 \times R = (1 \text{ A})^2 \times (0 \text{ ohm}) = 0$ watts.
A superconducting magnet requires no power to produce a magnetic field.
In contrast, a normal magnet requires a significant amount of power to produce a magnetic field. In the examples above, the normal magnet required 1000 kW whereas the superconducting magnet required 0 W.

5a. The superconductor must be maintained at low temperature so that it will be in the superconducting state.

5b. No power is needed to cause current to flow through the superconductor because its resistance is zero. Power is only needed to cause current to flow through the normal conductor. If the measured resistance of the graphite "non-superconducting" wire is 100 kohms, then the power needed is: $P=I^2 \times R = (1 \text{ A})^2 \times (100 \text{ kohm}) = 100 \text{ kW}$ or 10^5 W. So it still takes a quite a bit of power to operate our "superconducting" magnet, although the amount of power is about an order of magnitude less than that required to operate a non-superconducting magnet.

5c. Add what is called a "persistent switch." The switch consists of a superconductor that shorts the ends of the superconducting coil together when current is flowing in the superconducting coil. When the switch is closed, the current flows in a totally superconducting zero-resistance circuit - so that the power supply can then

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be turned off - but the electrical current continues to flow in the superconducting magnet. The switch in this drawing was made by wrapping copper wire around and under the head of a brass fastener. The copper wire is between the head of the fastener and the paper; the bendable parts of the fastener are spread out under the paper.

In reality, a persistent switch is not made out of moveable parts. It consists of superconductor with a heater wrapped around it. When the heater is on, the superconductor in the switch is at a temperature above the superconducting transition temperature, so is normal – it has a significant electrical resistance. In this situation, the switch is open – no current flows through the switch because the resistance of the switch is much higher than the resistance of the superconductor magnet.

When the heater is off, the superconductor in the switch is superconducting because it is now at a temperature below the superconducting transition temperature – it has no electrical resistance. In this situation, the switch is closed – so current will flow through the switch. Therefore, the current passing through the superconductor magnet will now pass through the switch instead of through the power supply because the electrical resistance of the leads leading to the power supply is much greater that the electrical resistance of the superconducting switch.

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Addendum to References

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National Lightning Safety Institute
http://www.lightningsafety.com

Lightning Detection from Space
http://thunder.nsstc.nasa.gov/primer/

Breakdown field in clouds
http://spike.geophys.washington.edu/Surface/Atmospheric/lightning/initaition.html

Lightning and cloud electrification
http://spike.geophys.washington.edu/Surface/Atmospheric/lightning/lightning.html

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Creative Drama A: Conductor vs. Semiconductor vs. Insulator Addendum

One of the reasons for this creative drama is to dispel a common misconception concerning insulators, semiconductors, and conductors. It is commonly believed that electrons move easily in a conductor, move more slowly in a semiconductor, and move even slower in an insulator. This is not true. In fact, in some semiconductors, such as GaAs, electrons move more easily (have higher mobility) than in a metal such as copper. Students should learn that it is the density of free (mobile) electrons n that differentiate these classes of materials. In a conductor, $n \sim 10^{22}$ cm^{-3}, whereas in a semiconductor, typical values for n are 10^{15} - 10^{19} cm^{-3}, and in an insulator, $n < 10^{10}$ cm^{-3}.

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A 2.

Tell the students that they will now perform a more sophisticated creative drama simulation of a semiconductor and show first how electron flow can contribute to the electrical conductivity of semiconductors. Next a simulation of hole conductivity will be performed, where holes, the absence of electrons, are responsible for the electrical conductivity.

To illustrate electron flow, arrange the students in the array shown in figure A2-1.

- Initially, of the students in row 1, only the student in the middle column has a ball that can move. (This ball should be different than the balls that don't move.)
- This student in row 1 then passes the ball to a student without a ball in row 2.
- This student in row 2 then passes the ball to a student without a ball in row 3.
- This student in row 3 then passes the ball to a student without a ball in row 4.
- This student in row 4 then passes the ball to a student without a ball in row 5.
- Then the battery person takes the ball from the person in row 5 back to a student without a ball in row 1.

As shown in the figure, the position of the free electron (ball) drifts down the lattice of atoms (students) until it reaches the battery terminal, where the battery then moves the electron back to the other side of the lattice.

In a semiconductor that is an electron conductor, there are many bound electrons, few mobile (free) electrons and many open spaces (available energy states) to which electrons can move.

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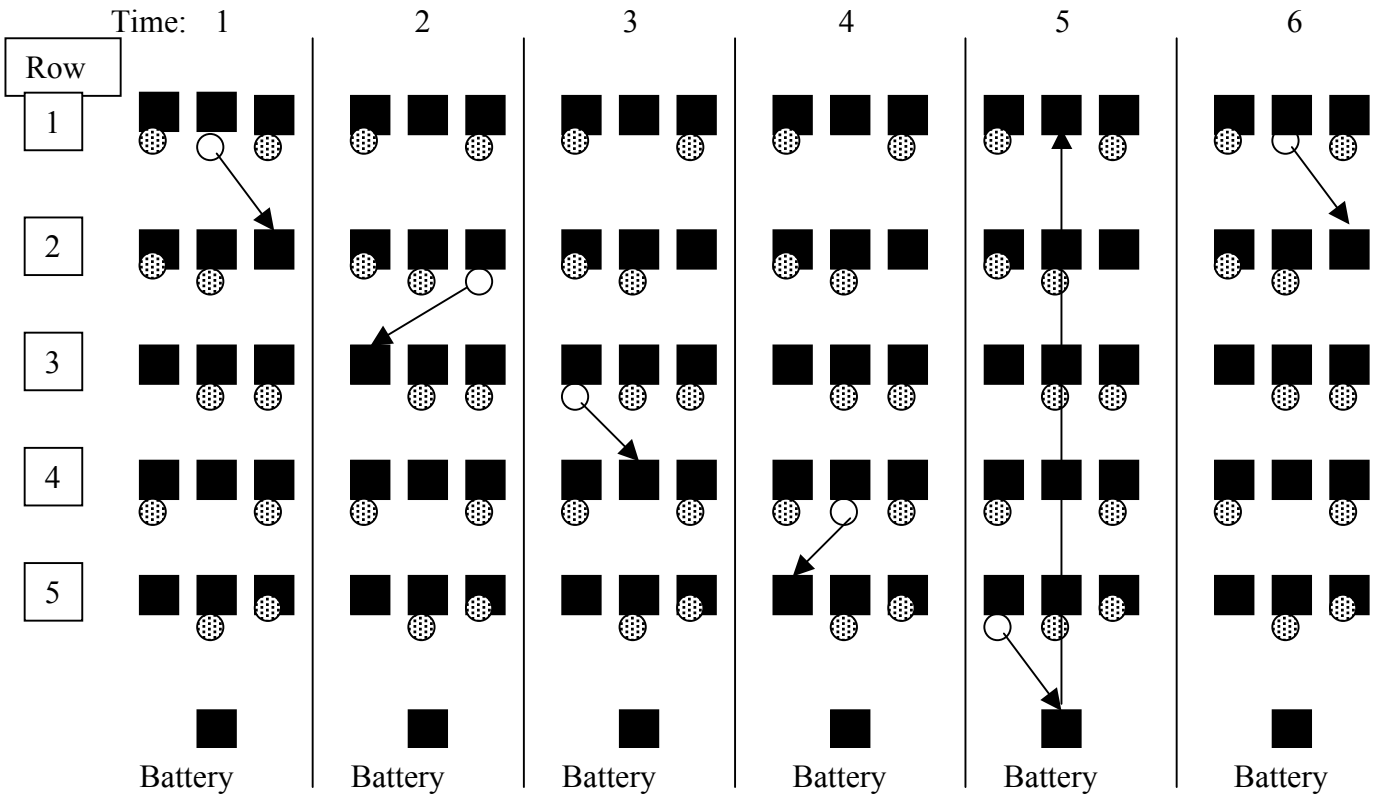

Bound electron

Free electron

Atom

Figure A2-1

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To illustrate hole flow, arrange the students in the array shown in figureA2-2.

- Initially, only one student in row 5 does not have a ball. One student in row 4 then passes their ball to the student in row 5.
- Now that student in row 4 does not have a ball. A student in row 3 then passes their ball to that student in row 4.
- Now that student in row 3 does not have a ball. A student in row 2 then passes their ball to that student in row 3.
- Now that student in row 2 does not have a ball. A student in row 1 then passes their ball to that student in row 2.
- Then the battery person takes the ball from a student in row 5 back to that student in row 1.

Note that the position of the hole, represented by the student without the ball, moves in the opposite direction to the ball motion: the electron (ball) moves in one direction, the hole (student without a ball) moves in the opposite direction.

In a semiconductor that is a hole conductor, there are only a few spaces (vacancies) available for electrons to move to. As shown above, the net effect of the movement of an electron is a net flow of a hole in the opposite direction.

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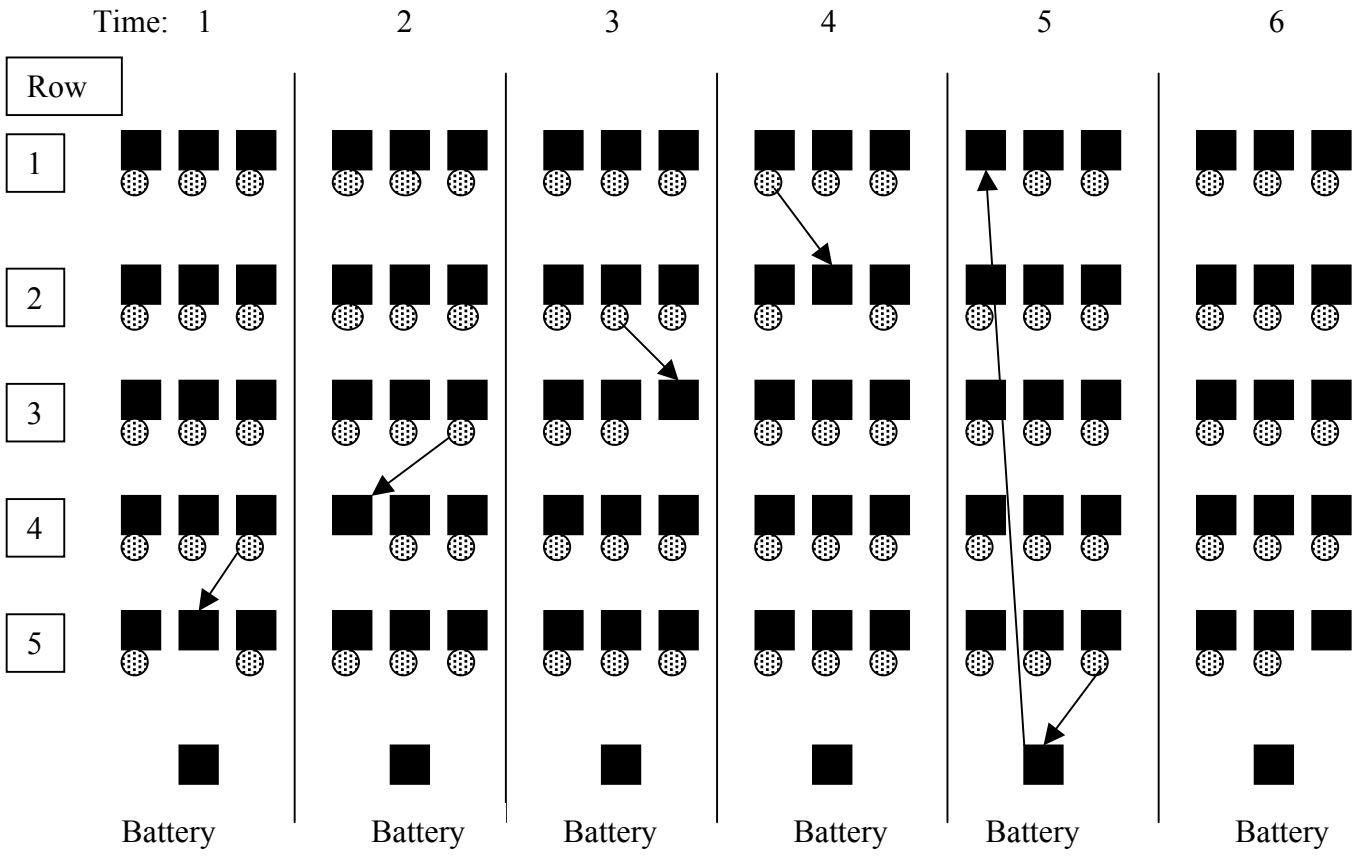

Loosely bound electron

Atom

Figure A2-2

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Assessment:

1. For the semiconductor with electrical conductivity due to electron flow, make a plot or histogram of the charge versus position for the electrons in each row for each time shown in figure A2-1.

2. For the semiconductor with electrical conductivity due to hole flow, make a plot or histogram of the charge versus position for the electrons in each row for each time shown in figure A2-2.

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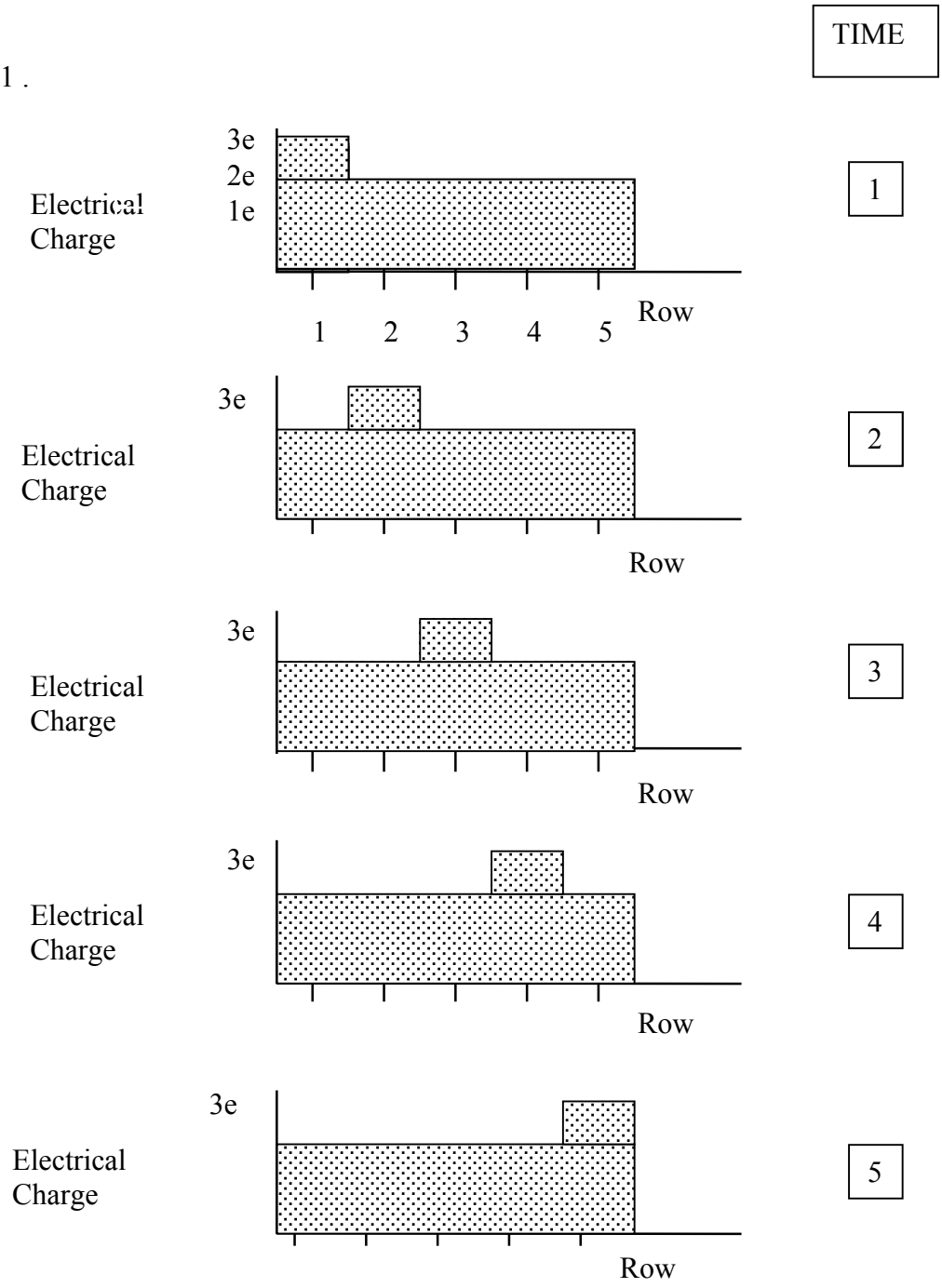
Answer:

1 .



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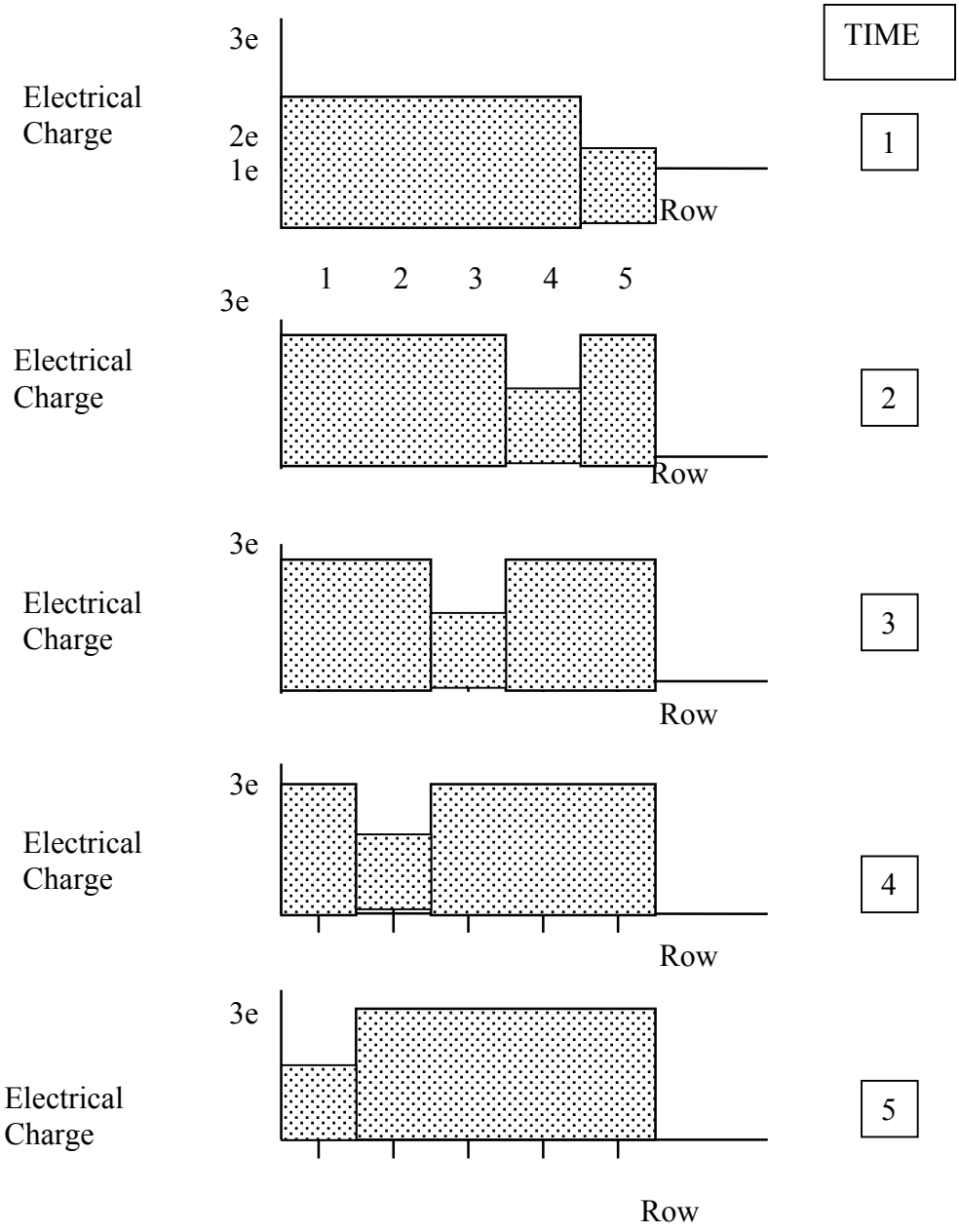

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